Precalculus Learning Goals - Week 1

This week is devoted to a section on **Algebra Review**. The general goals for this section are as follows. At the end of this section, students should be able to:

- *Creatively and correctly apply the rules of algebra, and justify steps taken.*
- *Recognize and disprove an incorrect application of the rules of algebra.*
- *Apply enlightened guessing to follow common algebraic procedures.*

More specifically, you should be able to:

- Perform arithmetic operations on real numbers and variables.
- Recognize when parentheses are needed to make an expression unambiguous.
- Apply rules of algebra (arithmetic, order of operations, distributive law, rules of exponents, etc.) to rewrite expressions.
- Factor quadratics.
- Explain what the solution set to an equation or inequality is.
- Solve simple algebraic equations through algebraic operations and by dividing into cases (including equations involving absolute value).
- Solve simple algebraic inequalities through algebraic operations and by dividing into cases (including inequalities involving absolute value).
- Use the "Zero-Factor Theorem" to solve equations through factoring.
- Check solutions to an equation or inequality.
- Use interval notation to define sets of real numbers.
- Explain the difference between intersections and unions of sets.
- Identify mistakes in a series of algebraic steps.
- Compute the distance between points in the plane.
- Identify sets of points in the plane described by words, sets, or equations.
- Transition between verbal and symbolic descriptions of circles.

Sample Problems. Here are some sample problems, of the type that you would do to demonstrate that you've learned the material. These are not the only types of problems you may see – they're just a sample.

- Write the set $\{x \mid x \neq 5\}$ using interval notation.
- Simplify $\left(\frac{-32}{w^{10}}\right)^{-\frac{3}{5}}$.
- Simplify $\frac{x^{-1}+y^{-1}}{(xy)^{-1}}$.
- Solve for c: $E = mc^2$.
- Solve for m: $(m-2)^2 = m^2 + 5$.
- Find the solution set to the inequality $|x+3| > 15$.
- Is 28 in the solution set of $|x-2| - |x+7| < 32$?
- Write the set $(-\infty, 2) \cap (1, 2]$ as a single interval, using interval notation. Sketch a picture of this interval and describe it in words.
- Factor $x^2 - 6x - 16$.
- Solve for x: $3 + x = \frac{5-x}{x}$.
- Factor $4x^2 - 25y^2$.
- Find the distance between the points $(1, 2)$ and $(4, 6)$.
- Find the equation of a circle that has its center at $(1, 2)$ and passes through $(4, 6)$.
- Heres an expression: $\frac{5-x^2}{x+y}$. Add parentheses in two different ways to make it mean two different things.

- Find and correct the mistake: $\sqrt{x}/3 = 2 \implies x/9 = 2 \implies x = 18$.
- John says "I took 100 minus 8 plus 17," and Bill says "I took 100 minus 8 plus 17." They got different answers, but they're both correct. What's going on?
- Is there a difference between $(a^b)^c$ and $a^{(b^c)}$? If so, illustrate the difference using numbers.
- How many points (x, y) are there that are 2 units from the origin and such that $x = 2y$?
- Find an equation that has $-\pi$ and 12 as solutions.
- Heres an equation and "solution:" $\sqrt{x} + 6 = 2 \implies \sqrt{x} = -4 \implies x = 16$. But $\sqrt{16} + 6 = 10$, not 2. What went wrong?